# Ant Colony Optimization-based Method for Managing Industrial Influents in Wastewater Systems

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The effectiveness of wastewater systems with high-industrial effluent input requires avoiding temporary overloads due to influent volumes and/or pollutant loads that exceed the system's treatment capacity. The multiagent paradigm is shown to be a suitable methodology for managing all information related to the state of the entire system to apply optimal influent assignment criteria. However, to be efficient, this methodology requires a prioritization process to assign priorities to different influent classes. A novel approach is proposed for solving this complex issue using a combinatorial optimization procedure with multiple constraints that is implemented when the treatment system lacks the capacity to accept all of its influent. The metaheuristic method applied is an ant colony optimization-based method, and the solution is achieved using two distinct algorithms with different processes for updating the pheromone trail. The results illustrate their usefulness, even when industrial effluents exhibit large fluctuations in their pollutant concentrations. © 2011 American Institute of Chemical Engineers AIChE J, 58: 3070–3079, 2012

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# Introduction

The management of inputs to wastewater treatment plants (WWTPs) is a key issue for ensuring the high quality of the performance of WWTPs. This issue has special importance for WWTPs that treat wastewater with a high percentage of industrial influent due to the high variability of wastewater quality and quantity.

In WWTPs that use a biological treatment, the management procedures must also be able to smooth flows and loads to protect the microbial community that treats the pollutants. In addition, overloads of industrial effluents that might alter the treatment efficacy should be avoided. 1–3

Until now, the most commonly implemented procedure for controlling WWTP influent was based on the establishment of fixed limits for different types of input streams.<sup>4,5</sup> According to this procedure, each stream was assigned an upper limit that was established in accordance with the capacity of the treatment plant and the number and properties of the industrial effluents. Although this approach may be useful for avoiding overloads, it is not useful for coping

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with dynamic variations between different influents arriving at the WWTP. As a result, this process may undermine the ability of the plant to generate optimal profits. To attain optimal profitability, it is necessary to treat different inputs with diverse characteristics keeping two objectives in mind: to ensure the optimal capacity of the WWTP at any time and to treat the industrial effluents according to their relative urgency.

Several methods for managing industrial wastewaters can be found in the literature. One approach considers the feasibility of reducing industrial wastewater flow rates by minimizing fresh water consumption or maximizing water-reuse in the process industries. In this context, several authors have based their proposals on the water pinch analysis proposed by Wang and Smith, which has been combined with mathematical programming. Alva-Argáez et al.4 introduced a decomposition of a mixed-integer nonlinear programming problem (MINLP) into a sequence of mixed-integer linear programming problems (MILP) to find the network configuration that minimizes the overall demand for freshwater at minimum total cost. Jödicke et al.7 modeled a MILP to minimize total costs, including operation and investment costs. Prakotpol and Srinophakun<sup>8</sup> proposed a genetic algorithm toolbox for water pinch technology. Al-Redhwan et al.9 addressed the problem of uncertainty while optimizing water

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networks in process industries with a three-step methodology. The first step is to develop a deterministic optimization model, the second is a sensitivity analysis, and the third is developing a stochastic formulation. Moreover, Bandyopadhaya<sup>5</sup> developed a methodology to target the minimum freshwater requirement, maximum water reuse and minimum wastewater generation simultaneously. Liu et al. <sup>10</sup> proposed regeneration reuse for water-using networks with multiple contaminants

A second approach considers problems related to end-ofpipe wastewater management. After, Wang and Smith<sup>11</sup> defined a distributed effluent treatment systems, Kuo and Smith<sup>12</sup> introduced multiple contaminants. Using the same framework, Galan and Grossmann<sup>13</sup> developed a nonlinear programming (NLP) and MINLP model to identify the connections between the treatment technologies and their corresponding flow rates and compositions to discharge at minimum total cost while fulfilling the composition regulations. Their procedure solves a relaxed linear programming LP model and uses the solution as the starting point of the NLP model. Lee and Grossmann<sup>14</sup> applied a global optimization algorithm by considering a nonconvex generalized disjunctive programming (GDP) technique using principles based on a reformulation-linearization technique (RLT) to find the minimum cost. Meyer and Floudas<sup>15</sup> formulated wastewater treatment industries as a generalized pooling by applying a novel piecewise linear RLT formulation to find the global optimum. Additionally, Karuppiah and Grossmann<sup>16</sup> developed a deterministic spatial branch and contract algorithm by using a GDP to solve the choice of treatment technologies.

The authors proposed an alternative technique based on the use of the multiagent paradigm. <sup>17</sup> In this approach, each industrial activity that generated an influent to the WWTP was considered an entity (or agent) with its own schedule that may not be known by other agents. The WWTP was considered an agent as well. An interaction procedure between all of the agents was defined at each working step with the aim of finding to determine the optimal working conditions for the entire system at each decision step. Under this multiagent paradigm approach, the interactions among all agents in the system could be incorporated.

After defining the structure of the agents and their interrelationships, 18 a bottleneck was identified based on the concordance between the characteristics of industrial effluents generated and those of the industrial inputs that could be accepted by the WWTP. Problems arose when the total volume of industrial effluent generated exceeded the WWTP capacity, whether measured in terms of total flow or in terms of the treatment capacity for specific pollutants or both. For this reason, a new agent was defined with the aim of implementing a prioritization process based on the state of different agents in the system. This new coordinating agent received information on available WWTP capacity from a WWTP agent and information on industrial wastewater generation from industry agents and was able to prioritize discharges when the WWTP did not have enough capacity to admit all of the industrial volume and/or pollutant loads generated.

This article proposes an algorithm for such prioritization processes that aims to maximize the overall volume and pollutant loads discharged from industrial activities while preventing overloads to the WWTP. The process is considered an NP-Hard problem with characteristics similar to the well-known knapsack problem, which involves combinatorial

optimization in the presence of multiple constraints. In the abstract, the treatment system corresponds to the knapsack, and the multiple constraints correspond to the volumes and loads of pollutants admissible for treatment. In this context, an optimal solution can be found using an ant colony optimization (ACO) algorithm. <sup>19,20</sup>

An ACO algorithm can solve combinatorial optimizations with a stochastic procedure. The search for a solution is inspired by the behavior of real ants that use pheromone trails to guide their exploration; additionally, virtual ants can be helped by heuristic information. Zlochin et al.<sup>21</sup> analyzed and compared ACO with other methods for combinatorial optimization. Gutjahr<sup>22</sup> demonstrated the convergence of ACO algorithms to global optimal solutions.

The first ACO algorithm, known as the ant system (AS), was proposed by Dorigo<sup>19</sup> in 1996. AS had three versions: ant density, ant quantity and ant cycle processes. The first two were quickly abandoned because the ant cycle process gave the best results. The ant cycle process is the basis for all subsequent AS versions. There is a noted tendency of such models to add penalties when updating the pheromone trail under problem constraints. The penalties are implemented by the constraint-handling method described by Michalewicz and Schouenauer In 1996. The penalties are set based on the degree of constraint violation. As such, transforming the constrained problem into an unconstrained problem is favored because this technique expands the likelihood of finding a good solution.

To our knowledge, this is the first article to address the problem of industrial wastewater discharges with an ACO methodology that uses a cost function related to the composition of feasible discharges. This method can be used to generate a solution that addresses WWTP requirements on inputs and the outputs from industrial activities.

This proposal for a combinatorial optimization technique considers two different ACO versions based on the ant cycle process; both use local improvement algorithms for the constructed solutions. Specifically, the versions have different penalties in the construction phase of the solution. The first, the sP version, has a simple penalty related to the volumes. The second, the gP version, has a global penalty with multiple factors related to the volumes and the pollutant loads.

Our aim is to demonstrate this new approach for industrial wastewater discharges. The proposed method differs from other optimization approaches in that it constructs a solution from data related to the interactions of the system in question. That is, the solution is constructed based on the requirements of the system (particularly industrial activities and WWTP requirements) in real time by taking into account the internal procedures of each elementary system.

The article is structured as follows. The Materials and Methods section defines the influent assignment cycle and the prioritization process. Then it presents the combinatorial optimization technique, including the problem statement and specifications for the two versions of the ACO algorithm. The Results and Discussion section presents the results obtained in the case study. First, this section shows the protocol used to perform the simulations. Then it defines the scenarios and the calibration procedure for the sP and gP versions, including the values obtained for their respective parameters. At the end of the section, it presents the results obtained from the simulations involving the calibrated sP and gP model versions for all proposed scenarios. Finally, the conclusions section finishes the article.

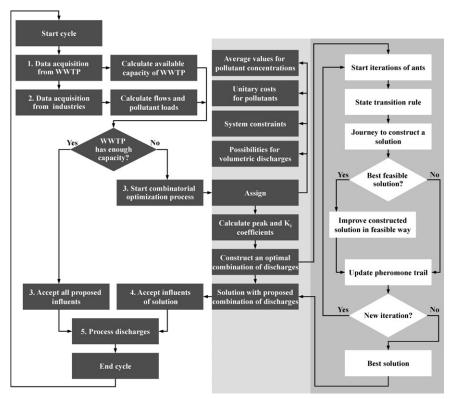


Figure 1. Decision cycle for the coordinating agent.

## **Materials and Methods**

## Influent assignment cycle

The procedure linking authorized flows to industrial activities is based on the following decision cycle rules.

Figure 1 depicts decision rules for each cycle; note that the decision rules follow steps 1 to 5.

1. Data acquisition from the WWTP agent.

In this step, the status of the WWTP is determined. Specifically, its overall flow and the total pollutant loading capacities over the next 24 h are derived.

2. Data acquisition from the industry agents.

In this step, the proposed flows and their pollutant compositions resulting from industrial activities over the next 24 h are determined.

- 3. If the WWTP does have not enough capacity to accept all industrial wastewaters, then a combinatorial optimization process for discharges is executed; otherwise, the WWTP accepts all proposed influents.
  - 4. Accept the industrial discharges authorized in step 3.
  - 5. Process discharges.

# Combinatorial optimization process

Problem Statement. A system of industrial activities Indi with  $i \in \{1,...n\}$  is considered. For each activity, there is a retention tank used to store the resulting wastewater volume  $L_i$  that has different possible volumetric discharges  $V_i^l$ ,  $j \in$   $\{0,\ldots,l_i\}$  that are selected as a multiple of a number (in this case, 100) such that 100  $j = V_i^j$  and 100  $l_i = L_i$ . Stored wastewater may have the following pollutants: total suspended solids (TSS), biochemical oxygen demand (BOD), chemical oxygen demand (COD), total nitrogen (TN), and total phosphorous (TP). Together, these values are denoted as a set of pollutants  $x_r$  with (TSS, BOD, COD, TN, TP) =  $(x_1, x_2, x_3, x_4, x_5)$  and concentrations  $C_i^r$ , r = 1,...,5. These wastewater discharges compose the inflow to the treatment plant. The WWTP has an admissible volume V and admissible concentrations of pollutants  $C^r$ , r = 1,...,5. The existing laws over the characterization of WWTP outputs have conditioned the selection of the set of pollutants to consider.

The following variables are assigned:

- (a) A unitary cost  $P^r$  and a weight  $w^r$  are assigned to each pollutant  $x_r$  according to the requirements for its treatment. Both of these values are considered individually for all industrial activities.
- (b) A peak coefficient  $T_i^r = f(C_i^r, \bar{X}_i^r)$  as defined in Table 1 is assigned to each industrial activity. Note that  $\bar{X}_i^r$  is the preset average value of the pollutant concentrations for each industrial activity. The values of the ratio  $C_i^r/\bar{X}_i^r$  are given in terms of intervals. For each interval,  $T_i^r$  has a specific value. An increase in each coefficient value increases the degree of pollutant overloads relative to their expected values. If any pollutant concentration in an industrial wastewater retention tank exceeds 112% of its average expected value, the peak

Table 1. Values and Additional Rules of the Peak Coefficient

$C_i^r/ar{X}_i^r$	0-1.11	1.12–1.25	1.26–1.50	1.51–1.75	1.76-2.00	2.01-3.00	3.01-4.00	4.01-5.00	> 5.00
$T_i^r$	1	1.1	1.2	1.5	1.7	2.0	2.5	3	Value of $C_i^r/\bar{X}_i^r$ maximum 10

Source: Government of Catalonia DOGC 4015.

Additional rules: If  $X_i^r = 0$  and  $C_i^r = 0$ , then  $T_i^r = 1$ ; on the other hand, if  $X_i^r = 0$  and  $C_i^r > 0$ , then  $T_i^r = 10$ .

coefficient increases its cost, which becomes greater for greater overloads.

(c) A coefficient  $K_i$  is assigned to each industrial activity by considering the saturation degree of the discharge  $V_i^j$ relative to the admissible volume V in the WWTP. The authors defined this coefficient using an increasing continuous piecewise function with domain [0,V] and range [0,1]as follows

$$K_i = \frac{V_i^j}{2V}$$
 if  $0 \le V_i^j \le V/2$  and  $K_i = \frac{3V_i^j}{2V} - \frac{1}{2}$  if  $V/2 < V_i^j \le V$ 

Objective Function and Constraints. The objective is to maximize a global cost function Z, as defined in Eq. 2

$$Z = \sum_{i=1}^{n} \sum_{j=0}^{l_i} y_i^j V_i^j \left( v + \psi K_i \sum_{r=1}^{5} C_i^r P^r T_i^r \right)$$
 (2)

Note that  $y_i^j$  are the binary decision variables that take values equal to 1 if  $Ind_i$  discharges  $V_i^J$ , and 0 otherwise. The coefficient v is the cost per unit of volumetric discharge, and the coefficient  $\psi$  is the weight of the cost of pollutant loads. Equation 2 is based on regional costs for industrial discharges<sup>29</sup> that includes a volumetric cost and a specific cost for pollutant loads. Accordingly, the selected values are v = 0.1314 and  $\psi = 1.5$ . The volumetric cost of Eq. 2 is evaluated by the first term of the equation in the same way for all wastewaters. The cost of pollutant loads is evaluated by the second term in a specific way for each wastewater, which depends on the characteristics of the discharges.

The function Z is constrained by

$$\sum_{i=1}^{n} \sum_{j=1}^{l_i} y_i^j V_i^j \le V \quad \text{(volume constraint)} \tag{3}$$

$$\sum_{i=1}^{n} \sum_{j=0}^{l_i} y_i^j V_i^j C_i^r \le VC^r, r = 1, ..., 5 \quad \text{(pollutant load constraints)}$$

$$\sum_{i=0}^{l_i} y_i^j = 1, i = 1, ..., n \quad \text{(decision variable constraints)} \quad (5)$$

If a solution satisfies all constraints, it is considered a feasible solution; otherwise, it is considered nonfeasible.

# The ACO algorithm

The ACO algorithm used in this work for combinatorial optimization was based on the ant cycle procedure, which comprises two phases. The first one is the main algorithm used to construct the solution. The second one is aimed at the local improvement of the constructed solution.

Construction of the Solution and Penalties on the Pheromone Updating Process. The algorithm uses a population of M ants, each of which is initially placed in a random location. Each ant constructs a solution by applying a probabilistic rule called the state transition rule in its journey around the space of solutions represented by a bipartite graph G = (N, E), where  $N = \{Ind_i\} \cup \{V_i^l\}$  is the set of nodes, and E is the set of edges. Each edge connects each industrial activity with its possible discharges. The edge (i, j) means that  $Ind_i$  discharges  $V_i^j$ .

The state transition rule for the ants is represented by the probabilistic equation from Dorigo, 19 which is adapted in Eq. 6 with heuristic information appropriate to our case

$$p_{ij}^{m}(t) \frac{\left[\tau_{ij}(t)\right]^{\alpha} \left[V_{i}^{j} \sum_{r=1}^{5} w^{r} C_{i}^{r} T_{i}^{r}\right]^{\beta}}{\sum_{m=1}^{M} \left[\tau_{ij}(t)\right]^{\alpha} \left[V_{i}^{j} \sum_{r=1}^{5} w^{r} C_{i}^{r} T_{i}^{r}\right]^{\beta}}$$
(6)

where  $p_{ii}^m(t)$  is the probability that the *mth* ant chooses volume  $V_i^j$ ,  $\tau_{ij}(t)$  is the trail of pheromone at time t,  $\alpha$  is the importance of the pheromone,  $V_i^j \sum_{r=1}^5 w^r C_i^r T_i^r$  is the heuristic information used to address discharges with higher loads of pollutants, and  $\beta$  is the importance of this heuristic information.

The ants search for a solution iteratively. After each iteration, all of the ants that have participated in the search for a solution and that have completed a tour update the pheromone trail. The updating procedure generates large amounts of pheromone on the edges when a feasible solution is constructed and smaller amounts when a nonfeasible solution is constructed. The amount of pheromone is related to the quality of solution.

The trail of pheromone is updated according to Eq. 7

$$\tau_{ij}(t+1) = \rho \tau_{ij}(t) + \Delta \tau_{ij}(t) \tag{7}$$

where  $0 \le \rho \le 1$  is a coefficient for the pheromone evaporation such that  $1 - \rho$  represents the evaporation of trail, and

$$\Delta \tau_{ij}(t) = \sum_{m=1}^{M} \Delta \tau_{ij}^{m}(t) \quad \text{with}$$

$$\Delta \tau_{ij}^{m}(t) = \begin{cases} QZ_{m} \ penalty_{m} & \text{if the } m\text{th ant uses the edge } (i,j) \\ 0 & \text{otherwise} \end{cases}$$
(8)

Q is a positive number,  $Z_m$  is the solution obtained by the mth ant in its path, and  $penalty_m$  is a value that varies according to the version of the algorithm, as follows.

(a) For the sP version, a simple penalty is related to the total volume

$$penalty_m = \begin{cases} \left(\frac{V}{TV_m}\right)^a & \text{if } V < TV_m \\ \left(\frac{Z_m}{T^*}\right)^b & \text{if } V \ge TV_m \end{cases}$$
(9)

where  $TV_m = \sum_{i=1}^n V_{i,m}$  is the total volume of the influent received by the WWTP as per the path of the mth ant, a is the importance attached to whether the total volume obtained in the path exceeds the admissible volume V of the WWTP,  $Z^*$  is the best obtained solution in the iteration, and b is the importance attached to the ratio between the obtained solution and the best solution.

Note that version sP generates smaller amounts of pheromone when  $TV_m$  exceeds V, and  $Z_m$  when is far from  $Z^*$ .

(b) The gP version has a global penalty with multiple factors related to the total volume and total pollutant loads

$$penalty_{m} = \begin{cases} \left(\frac{V}{TV_{m}}\right)^{a} \left(\frac{Lo^{1}}{Lo_{m}^{1}}\right)^{a_{1}} \left(\frac{Lo^{2}}{Lo_{m}^{2}}\right)^{a_{2}} \left(\frac{Lo^{3}}{Lo_{m}^{3}}\right)^{a_{3}} \left(\frac{Lo^{4}}{Lo_{m}^{4}}\right)^{a_{4}} \left(\frac{Lo^{5}}{Lo_{m}^{3}}\right)^{a_{5}} & \text{if } V < TV_{m} \text{ or } Lo^{r} < Lo_{m}^{r} \text{ for some } r = 1, ..., 5 \\ \left(\frac{Z_{m}}{Z^{*}}\right)^{b} & \text{if } V \geq TV_{m} \text{ and } Lo^{r} \geq Lo_{m}^{r} \text{ for all } r = 1, ..., 5 \end{cases}$$

$$(10)$$

where

If  $Lo^r < Lo_m^r$  then  $a_r = 2$ ; otherwise  $a_r = 0$ , r = 1,...,5. If,  $V < TV_m$  then; a = 2; otherwise, a = 0.

 $Lo^r = VC^r$  indicates the admissible loads of pollutants  $x_r$ for the WWTP,  $Lo_m^r = \sum_{i=1}^n C_i^r V_{i,m}$  is the load of pollutants  $x_r$  according to the path of the mth ant, and  $a_r$  is the importance attached to whether the pollutant loads obtained in the path exceed the admissible pollutant loads for the WWTP.

Note that version gP generates smaller amounts of pheromone when  $TV_m$  exceeds V, when some  $Lo_m^r$  exceeds  $Lo^r$ , and when  $Z_m$  is far from  $Z^*$ .

Local Improvement of the Constructed Solution. For each iteration, the best constructed solution  $Z^*$  can be processed using two local improvement algorithms.

The first algorithm aims to improve  $Z^*$  through the partial discharge of wastewaters that are still stored in retention tanks  $\sum_{i=1}^{n} L_i - V_i^*$ , where  $V_i^*$  is the volume of discharge for Indi in the constructed solution. If a new volume of discharge provides an improvement to a feasible solution making a new feasible solution, this new volume is added and a new feasible solution is considered; otherwise, the new volume is not added, and a new solution is not considered. When the best feasible solution is achieved, the new solution  $Z^*$ ; replaces the old one.

The second algorithm improves the value of  $Z^*$ ; by searching in its neighborhood. A solution is interpreted as a vector of n discharges. The algorithm performs an increase in one component of the solution, a decrease in another and no changes in the others. Note that such vectors describe the neighborhood of a solution, which has the cardinal number n(n-1). If a change in a component to its neighbor improves a feasible solution and makes a new feasible solution, this new component is included in the solution instead of the old component, and a new feasible solution is considered; otherwise, the new component is not included, and a new solution is not considered. When the best feasible solution is achieved, the new solution Z replaces the old one.

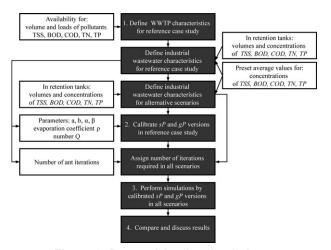


Figure 2. Protocol for the simulations.

## **Results and Discussion**

Figure 2 is a diagram of the protocol used to perform the simulations. First, the characteristics of the system under study are specified. Second, the sP and gP versions of the algorithm are calibrated. Third, the simulations are performed in all scenarios with the calibrated algorithm versions. Finally, the results obtained across all scenarios are compared and discussed.

# Defining the scenarios

Reference Case Study. The reference case study (Table 2), also called Scenario 1, consists of a WWTP and a set of 25 industrial activities that generate different wastewater compositions, which are defined by the set of pollutants  $x_r$ , r = 1,...5. For each activity, preset average concentrations  $\bar{X}_i^r$  for pollutants  $x_r$  are defined to correspond to the values expected during normal operation. Also, for each activity the values corresponding to data received in real-time are defined for the coordinating cycle of the storage volume of wastewater  $L_i$  and its pollutant concentrations  $C_i^r$ , r = 1,...5. The values  $\bar{X}_i^r$ ,  $L_i$  and  $C_i^r$  for Scenario 1 are based on data for some wastewater outputs from industrial activities in the region.

The absolute values of the differences between the values of  $C_i^r$  and their preset average concentrations  $\bar{X}_i^r$  are represented by  $\bar{D}_i^r$  (Table 3). The values that show a peak coefficient greater than 1 compose 17.6% of the values of the pollutant concentrations.

Alternative scenarios. To explore the behavior of the sP and gP versions under different conditions, Scenarios 2 and 3 were considered.

Scenario 2 included the same values for  $L_i$  as in Scenario 1. It differed from Scenario 1 in that it had higher values for  $C_i^r$  and  $\bar{D}_i^r$  with larger stored pollutant loads  $\sum_{i=1}^n L_i C_i^r$  (see Table 3). In addition, 75.2% of the pollutant concentrations exhibited peak coefficients greater than 1.

Scenario 3 had the same values for the pollutant concentrations  $C_i^r$  as Scenario 1; thus, both scenarios had the same values for the standard deviations,  $\bar{D}_i^r$  and peak coefficients. However, Scenario 3 differed from Scenario 1 in that it had a larger overall stored volume, implying a larger total value for the pollutant loads (Table 3). All industrial activities had an equal value for volumes stored  $L_i = 1000$ .

## Calibration procedure

Before applying the algorithm to determine an optimum solution with the maximum overall volume and pollutant loads, it was necessary to calibrate the values of the algorithm parameters that defined the ant cycle procedure for the sP and gP versions of the algorithm.

The calibration procedure was performed following the reference strategy. 19,26 All tests were performed with 500 iterations and 100 ants per iteration, and the results were averaged over 10 repetitions. The first iteration of each repetition started with the same value (100) of the pheromone trail on all edges to give all edges the same probability of being chosen by ants.

Table 2. Data from the Reference Case Study

i	$X_i^r$ (g/m <sup>3</sup> )					$C_i^r$ (g/m <sup>3</sup> )					
	$x_1$	$x_2$	$x_3$	$\chi_4$	$\chi_5$	$x_1$	$x_2$	$x_3$	<i>x</i> <sub>4</sub>	$\chi_5$	$L_i (m^3)$
1	200	150	1010	19	9	157	210	870	9	1	100
2	300	350	500	25	13	300	200	300	10	9	200
3	500	100	850	5	0	450	100	1050	5	0	500
4	750	500	850	40	0	650	500	950	30	0	2000
5	650	750	800	90	10	650	550	650	90	10	200
6	100	700	800	20	0	100	700	900	10	0	700
7	650	750	800	35	15	570	600	760	35	12	500
8	610	100	740	0	0	670	100	850	0	0	200
9	50	20	200	2	0	40	20	150	2	0	300
10	167	171	433	2 2	1	600	320	490	2	1	200
11	64	249	623	40	4	40	270	620	43	6	3000
12	300	415	670	5	1	550	495	570	13	2	400
13	950	188	375	12	4	125	135	520	4	1	300
14	350	200	500	15	2	375	180	340	7	0	100
15	180	95	810	30	1	200	550	800	23	4	700
16	300	800	940	35	6	150	350	540	38	2	2000
17	760	620	1150	5	0	260	420	550	15	0	500
18	160	360	580	90	7	180	310	480	40	5	500
19	565	690	950	20	1	365	390	550	10	2	100
20	600	750	800	10	15	500	500	700	2	10	100
21	600	750	800	10	15	612	450	680	10	0	500
22	700	1300	2325	20	10	498	700	810	50	12	100
23	500	600	950	50	15	506	670	820	50	5	400
24	500	800	1000	50	15	226	850	930	10	2	100
25	300	400	650	20	0	180	350	550	30	18	1000
					V	/WTP					
	$x_1$		$x_2$	$x_2$ $x_3$		$x_4$		$x_5$		$V (m^3)$	
$W^{r}$ (*) $P^{r}$ (**) ( $\mathcal{E}/\mathrm{Kg}$ ) $VC^{r}$ ( $\mathrm{Kg}$ )		0.08 0.3980 8400		0.11 0.5000 7800		0.11 1.1945 8400	0.	0.10 6145 .200	0.10 1.209 240	2	12000

<sup>(\*)</sup> Values based on weight factors from Nation Sanitation Foundation Water Quality Index. (\*\*) Values based on those provided by the Government of Catalonia DOGC 5288.<sup>29</sup>

Table 3. Data from Scenarios 1, 2 and 3

			Scen	ario 1	Scena	ario 2	Scenario 3	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
$L_{i}$	$\sum_{i=1}^{n} L_i \text{ (m}^3)$ $\bar{L}_i \text{ (m}^3)$	$\sum_{i=1}^{n} L_i \text{ related to } V (\%)$	14700	122.5	14700	122.5	25000	208.3
	$\bar{L}_i^{i=1}$ (m <sup>3</sup> )	$\sigma_{I}$	588	716.08	588	716.08	1000	0
$x_1$	$\sum_{i=1}^{n} L_i C_i^1 \text{ (Kg)}$ $\bar{C}^1 \text{ (g/m}^3)$ $\bar{T}^1$	$\sum_{i=1}^{n} L_i C_i^1 \text{ related to } VC^1 \ (\%)$	4274	50.9	8548	101.8	8954	106.6
	$\bar{C}^{1}$ (g/m <sup>3</sup> )	$\sigma_{C^1}$	358.16	211.02	716.32	422.04	358.16	211.02
	$ar{T}^1$	$\sigma_{T^1}$	1.09	0.32	1.79	1.18	1.09	0.32
	$ar{D}^1$	$\sigma_{D^1}$	140.16	195.67	363.12	282.19	140.16	195.67
$x_2$	$\sum_{i=1}^{n} L_i C_i^2 \text{ (Kg)}$ $\bar{C}^2 \text{ (g/m}^3)$	$\sum_{i=1}^{n} L_i C_i^2 \text{ related to } VC^2 \ (\%)$	5704.5	73.1	11409	146.3	9920	127.2
	$\bar{C}^2$ (g/m <sup>3</sup> )	$\sigma_{C^2}$	396.80	216.77	793.60	433.53	396.80	216.77
	$ar{T}^2$	$\sigma_{T^2}$	1.24	0.96	1.89	1.73	1.24	0.96
	$ar{D}^2$	$\sigma_{D^2}$	146.32	164.57	329.28	274.20	146.32	164.57
$x_3$	$ \sum_{i=1}^{n} L_{i}C_{i}^{3} \text{ (Kg)} $ $ \bar{C}^{3} \text{ (g/m}^{3}) $ $ \bar{T}^{3} $ $ \bar{D}^{3} $	$\sum_{i=1}^{n} L_i C_i^3 \text{ related to } VC^3 \ (\%)$	9975	118.7	19950	237.5	16430	195.6
	$\bar{C}^3$ (g/m <sup>3</sup> )	$i=1$ $\sigma_{C^3}$	657.20	219.84	1314.40	439.68	657.20	219.84
	$\widetilde{T}^3$	$\sigma_{T^3}$	1.02	0.05	1.54	0.34	1.02	0.05
	$ar{D}^3$	$\sigma_{D^3}$	204.00	304.09	570.56	325.75	204.00	304.09
$x_4$	$ \sum_{i=1}^{n} L_i C_i^4 \text{ (Kg)} $ $ \bar{C}^4 \text{ (g/m}^3) $ $ \bar{T}^4 $	$\sum_{i=1}^{n} L_i C_i^4 \text{ related to } VC^4 (\%)$	426.8	35.6	853.6	71.1	538	44.8
	$\bar{C}^{4}$ (g/m <sup>3</sup> )	$\sigma_{C^4}$	21.52	21.45	43.04	42.91	21.52	21.45
	$\widetilde{T}^4$	$\sigma_{T^4}$	1.13	0.33	1.79	1.25	1.13	0.33
	$ar{D}^4$	$\sigma_{D^4}$	9.60	12.66	21.60	24.91	9.60	12.66
<i>x</i> <sub>5</sub>	$ \sum_{i=1}^{n} L_i C_i^5 \text{ (Kg)} $ $ \bar{C}^5 \text{ (g/m}^3) $ $ \bar{T}^5 $	$\sum_{i=1}^{n} L_i C_i^5 \text{ related to } VC^5 (\%)$	61.1	25.5	122.2	50.9	102	42.5
	$\bar{C}^{5}$ (g/m <sup>3</sup> )	$\sigma_{C^5}$	4.08	4.99	8.16	9.98	4.08	4.99
		$\sigma_{T^5}$	1.48	1.81	1.94	2.20	1.48	1.81
	$ar{D}^5$	$\sigma_{D^5}$	3.84	5.06	5.92	7.69	3.84	5.06

In the first step, the values for a,b (Eqs. 9 and 10),  $\alpha$  and  $\beta$  (Eq. 6) were tested using a combinatorial procedure of the reference values<sup>19</sup>  $a=2, b\in\{2, 8\}, \alpha\in\{0,0.2,0.5,1,2,5\}$  and  $\beta\in\{0,0.2,0.5,1,2,5\}$  constants<sup>26</sup>  $\rho=0.99$  and Q=1000. The values for parameters  $a_1,\ldots,a_5=2$  (Eq. 10) were added for the gP version.

In the second step, the values for  $\rho$  (Eq. 7) and Q (Eq. 8) were tested individually with the reference values  $^{19}$   $\rho \in \{0.1,0.3,0.5,0.7,0.9,0.99\}$  and  $Q \in \{100,500,1000,5000\}$  and the values of a, b,  $\alpha$  and  $\beta$  that returned the best solution in the first step as constants.

Additionally, the number of ant iterations per algorithm repetition needed to achieve a stable solution was observed, and best solutions were found iteratively using the values a, b,  $\alpha$ ,  $\beta$ ,  $\rho$  and Q as constants that previously returned the best solution. This process was followed for Scenarios 2 and 3 as well.

The parameter values that offered the best average solutions were as follows.

- (a) For the *sP* version, a = 2, b = 2,  $\alpha = 1$ ,  $\beta = 0$ ,  $\rho = 0.9$  and Q = 1000, resulting in a cost  $\bar{Z} = 3002.86$ ,  $\sigma_Z = 5.91$  and  $Z_{\text{max}} = 3011.84$ .
- (b) For the *gP* version, a = 2,  $a_r = 2$ , b = 2,  $\alpha = 1$ ,  $\beta = 0.2$ ,  $\rho = 0.9$  and Q = 1000, resulting in a cost  $\bar{Z} = 3007.56$ ,  $\sigma_Z = 8.61$  and  $Z_{\rm max} = 3016.52$ .

The value obtained for the heuristic importance  $\beta=0$  in the sP version indicated that only the pheromone was considered in the state transition rule (Eq. 6). The value obtained for the importance of the pheromone was  $\alpha=1$ . Therefore, the value of the probability that the mth ant chooses volume  $V_i^j$  (Eq. 6) depends on the values of the process used to update the pheromone trail (Eq. 7). For this process using a simple penalty (Eq. 9), the penalty values achieved with the values a=2 and b=2 are beneficial in the search for a solution because of the amounts of pheromone generated on the edges. Moreover, with the addition of the local search algorithms, the heuristic information is no longer necessary to guide ants to achieve a good solution.  $^{20}$ 

The value  $\beta=0.2$  obtained for the gP version presented a small increase compared to that obtained under a simple penalty. For the state transition rule (Eq. 6) with  $\alpha=1$ , the value to some extent offset the decrease in the tendency to discharge larger pollutant loads, which was due to the multiple factors that affected the penalty values (Eq. 10) with a=2,  $a_r=2$ , b=2 and, consequently, the pheromone trail.

For both the sP and gP versions, the best values of  $\overline{Z}$  based on the tested values of the coefficient for the pheromone evaporation were achieved with  $\rho=0.9$ , thus, the evaporation rate was 0.1.

The tested values for Q resulted in very small differences across the solutions found. For the sP version, the solutions ranged from  $\overline{Z}=3002.02$  with  $\sigma_z=7.25$  to  $\overline{Z}=3004.01$  with  $\sigma_z=7.52$ , and for the gP version, they ranged from  $\overline{Z}=3002.94$  with  $\sigma_z=8.36$  to  $\overline{Z}=3005.61$  with  $\sigma_z=10.17$ .

The best solutions that were iteratively achieved using a simple penalty, presented a small improvement after 100 iterations for Scenarios 1 and 3, and after 150 iterations for Scenario 2. For Scenario 1, the evolution of the best solutions achieved using a global penalty was similar to that observed with a simple penalty. Scenarios 2 and 3 showed improvements up to 350 iterations. No improvements were observed after 500 iterations.

To compare the results without calibrating the algorithm parameters, the simulations were performed with the parameter values a=2, b=15,  $\alpha=1$ ,  $\beta=0.8$ ,  $\rho=0.99$  and Q=1000 that were successfully applied to solve another problem<sup>26</sup> that was also investigated as a knapsack problem. The values for  $a_1, \ldots, a_5=2$  were added for the gP version. For the reference case study, the returned solutions were  $\bar{Z}=2919.68$  with  $\sigma_Z=53.10$  under the sP version, and  $\bar{Z}=2921.54$  with  $\sigma_z=75.42$  under the gP version. For both the sP and gP versions, at least 5 algorithm repetitions achieved the solution at the first iterations. It should be noted that none of the solutions were found at the first iterations with the calibrated parameter values.

## Case study results and discussion

The simulations using the calibrated sP and gP versions were performed for all scenarios with 500 iterations and 100 ants per iteration, and the results were averaged over 10 repetitions. The initial value of the pheromone trail at the first iteration of an algorithm repetition was 100 for all edges. For each algorithm version, the solution to the cost of industrial wastewater discharges was related to the overall volume and the total loads of pollutants  $x_r$  of the discharged wastewaters. The path followed to achieve this solution provided the volumetric discharges according to each industrial activity in terms of values that corresponded to individual loads of pollutants.

Table 4 collects the data related to the solutions obtained using the sP and gP versions in the reference case study and the alternative scenarios. The average solutions for  $\bar{Z}$  according to both algorithm versions showed similar values for Scenarios 1 and 3, which were different from the Scenario 2 values.

For Scenario 1, both algorithm versions discharged the maximum admissible volume in the WWTP. Figure 3 shows the paths that derived the solutions. The tendency of the sP and gP versions to perform the discharges was similar to the profile of the initial storage volumes. The differences for behavior discharges were related to industrial activities with values of wastewater storage of  $L_i \leq 500$ . For these activities, the discharges returned by the gP version presented a decrease of 11.6% relative to those returned by the sP version when the values of peak coefficients were 1 for all pollutants, and presented an increase of 8.6% when the concentration values exceeded at least 112% their expected average values ( $T_i^r > 1$ ) for one or several pollutants.

For Scenario 2, the version gP provided the best solution. Both algorithm versions presented a decrease in the overall discharge volume related to that obtained in Scenarios 1 and 3 because of larger pollutant loads for the initial storage wastewaters (Table 3). For this scenario the values of  $T_i^r$  presented a large variability. The pollutant  $x_3$  played a dominant role in the assessment because of its high-concentration values and price. Figure 4 shows the different paths followed by both versions of the algorithm. The discharges returned by the gP version had a higher standard deviation (with  $\sigma_{gP}$ = 674.98) than those returned by the sP version (with  $\sigma_{sP}$  = 541.32). However, fewer industrial activities (76%) resulted in wastewater discharges by the gP version compared with those of the sP version (100%). For the industrial activities with initial volumes stored  $L_i$  < 2000, which composed 52.3% of the total wastewater stored in this scenario, the discharges returned by the gP version were 16.6% of the total wastewater discharged, and 61.3% lower than those returned by the sP version. Otherwise, for the industrial activities with  $L_i \geq 2000$ , the volumes returned under the gP version

Table 4. Results Based on the sP and gP Algorithm Versions

			sP		gP			
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3	
$\overset{-}{Z}$ (Cost) $\sigma_Z$		3002.86 5.91	2995.46 130.09	2450.47 21.16	3007.56 8.61	4098.33 211.79	2457.87 29.77	
$V_i^{(*)}$	$\sum_{i=1}^{n} V_i \ (\mathrm{m}^3)$	12000	6550	11970	12000	6070	11960	
	$\sum_{i=1}^{n} V_i \text{ related to } V (\%)$	100	54.6	99.7	100	50.6	99.7	
	$\sum_{i=1}^{n} V_i \text{ related to } \sum_{i=1}^{n} L_i (\%)$	81.6	44.6	47.9	81.6	41.3	47.8	
$x_1$	$\sum_{i=1}^{n} V_i C_i^1 $ (Kg)	3245.3	2849.3	4704.7	3235.6	3109.0	4739.9	
	$\sum_{i=1}^{n} V_i C_i^1 \text{ related to } VC^1 \ (\%)$	38.6	33.9	56.0	38.5	37.0	56.4	
	$\sum_{i=1}^{n} V_i C_i^1 \text{ related to } \sum_{i=1}^{n} L_i C_i^1 \text{ (\%)}$	75.9	33.3	52.5	75.7	36.4	52.9	
<i>x</i> <sub>2</sub>	$\sum_{i=1}^{n} V_i C_i^2 \text{ (Kg)}$	4748.3	4532.4	5246.3	4766.9	4201.5	5320.9	
	$\sum_{i=1}^{n} V_i C_i^2 \text{ related to } VC^2 \ (\%)$	60.9	58.1	67.3	61.1	53.9	68.2	
	$\sum_{i=1}^{n} V_i C_i^2 \text{ related to } \sum_{i=1}^{n} L_i C_i^2 \text{ (\%)}$	83.2	39.7	52.9	83.6	36.8	53.6	
<i>x</i> <sub>3</sub>	$\sum_{i=1}^{n} V_i C_i^3 $ (Kg)	8323.1	8382.4	8362.1	8321.8	8373.6	8273.8	
	$\sum_{i=1}^{n} V_i C_i^3 \text{ related to } VC^3 \ (\%)$	99.1	99.8	99.5	99.1	99.7	98.5	
	$\sum_{i=1}^{n} V_i C_i^3 \text{ related to } \sum_{i=1}^{n} L_i C_i^3 \text{ (\%)}$	83.4	42.0	50.9	83.4	41.9	50.4	
$x_4$	$\sum_{i=1}^{n} V_i C_i^4 \text{ (Kg)}$	381.9	433.1	280.5	372.2	429.3	288.6	
	$\sum_{i=1}^{n} V_i C_i^4 \text{ related to } VC^4 \ (\%)$	31.8	36.1	23.4	31.0	35.8	24.1	
	$\sum_{i=1}^{n} V_i C_i^4 \text{ related to } \sum_{i=1}^{n} L_i C_i^4 \text{ (\%)}$	89.5	50.7	52.1	87.2	50.3	53.6	
<i>x</i> <sub>5</sub>	$\sum_{i=1}^{n} V_i C_i^5 $ (Kg)	53.9	66.6	54.0	52.1	48.3	52.0	
	$\sum_{i=1}^{n} V_i C_i^5 \text{ related to } VC^5 (\%)$	22.5	27.7	22.5	21.7	20.1	21.7	
	$\sum_{i=1}^{n} V_i C_i^5 \text{ related to } \sum_{i=1}^{n} L_i C_i^5 \text{ (\%)}$	88.2	54.5	52.9	85.3	39.5	51.0	

<sup>(\*)</sup>Vi is the volume to be discharged from each industrial activity.

were 83.3% of the total volume discharged, and were increased by 28.4% relative to those returned by the sP version. This behavior can be observed in Figure 4 for activities 4, 11 and 16 with  $L_i \geq 2000$ . For these activities with  $T^3 = 1.6$ ,  $\sigma_{T^3} = 0.46$ , the storage loads of pollutant  $x_3$  were 48.5% of the total loads of  $x_3$  stored in Scenario 2; their dis-

charges under the gP version included 88.5% of the total load of  $x_3$  discharged and were increased by 43.8% relative to those returned by the sP version.

For Scenario 3, both versions returned similar values for the overall volume discharged and the total pollutant loads. Figure 5 shows the paths that produce these solutions. The

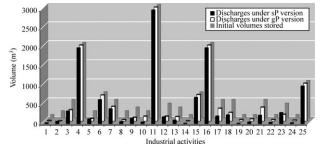


Figure 3. Wastewater discharges under the sP and gP versions for the reference case study.

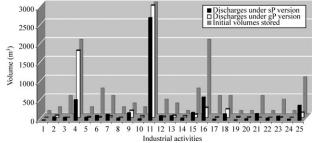


Figure 4. Wastewater discharges under the sP and gP versions for Scenario 2.

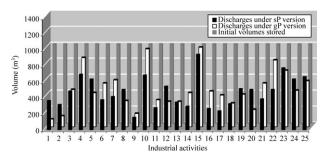


Figure 5. Wastewater discharges under the sP and gP versions for Scenario 3.

standard deviations of volumetric discharges from each industrial activity were smaller in the sP version (with  $\sigma_{sP}$  = 190.97) than in the gP version (with  $\sigma_{gP}=246.13$ ). The higher value of  $\sigma_{gP}$  was probably the result of multiple factors included in the global penalty. To account for the distribution of discharges presented with the same wastewater storage ( $L_i = 1000$ ) for all industrial activities, and the same values for the peak coefficient as Scenario 1, the resulting wastewaters were related to the pollutant  $x_3$  with a dominant role, followed by pollutants  $x_2$  and  $x_1$ . The industrial activities with values of  $C_i^3 < \vec{C}^3$ ,  $C_i^2 < \vec{C}^2$ ,  $C_i^l < \vec{C}^1$  for at least two of these pollutants achieved discharges with volumes  $V_i \leq \frac{1}{2}L_i$  under both algorithm versions. For industrial activities 1, 2, 9, 11, 13, 14, 16, 17 and 18, the initial concentrations of pollutants  $x_3$ ,  $x_2$  and  $x_1$  were lower than the values of their corresponding average pollutant concentrations in this scenario, for at least two of these pollutants. Otherwise, the industrial activities with  $C_i^3 > \bar{C}^3$ ,  $C_i^2 > \bar{C}^2$ ,  $C_i^I > \bar{C}^I$  for at least two of these pollutants presented discharges with volumes  $V_i > 500$ . Industrial activities 4, 15, 22 and 23 presented this behavior. For these activities, the initial concentrations of pollutants  $x_3$ ,  $x_2$  and  $x_1$ were larger than their corresponding average values in this scenario for at least two of these pollutants.

The initial values for the volume of wastewater stored and the concentration of the pollutant  $x_3$  exert large effects in all scenarios for the performance of discharges from industrial activities. For same values of wastewaters stored, the effects should also be related to the values of concentration of pollutants  $x_2$  and  $x_1$ .

## Conclusions

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The method proposed in this article for modeling industrial wastewater discharges is a new contribution to efforts aimed at preventing input overloads to WWTPs. Using the multiagent paradigm, a solution can be provided in real time based on system interaction data.

To solve the combinatorial optimization of wastewater discharges, two versions of an ACO algorithm, sP and gP, were used to address the dynamic variability between the industrial effluents by smoothing the effects of fluctuations in wastewater volumes and compositions. Both algorithm versions provided a cost solution aimed at maximizing overall wastewater discharges without overloading the WWTP. The value for overall wastewater discharges was obtained through an optimal combination of stored wastewaters from individual industrial activities, which favored large storage volumes.

For a set of industrial activities that did not have large overall volumes for stored wastewaters and did not exhibit large increases in pollutant concentrations relative to their expected average values (i.e., the reference case study), the sP and gP versions provided similar solutions. In this scenario, the behaviors of wastewater discharges were consistent with their profiles at initial storage. For a set of industrial activities that had large pollutant loads in stored wastewater and also exhibited large increases in pollutant concentrations relative to their expected average values (i.e., Scenario 2), the gP version achieved the best solution with respect to cost; the solution tended to favor discharge from fewer industrial activities and the large discharges from the large storage volumes. For a set of industrial activities with a large overall volume of stored wastewater and small increases in pollutant concentrations relative to their expected average values (i.e., Scenario 3), the sP and gP versions provided similar solutions, with similar discharge behaviors. The discharges presented slight differences from those provided when no large volumes were stored (i.e., the reference case study). The differences were due to the tendency to favor large pollutant concentrations when industrial activities had the same values for storage volumes.

For WWTP influents that may exhibit large fluctuations in pollutant concentrations, the discharge returned by the gP version, which had a global penalty with multiple factors related to volume and pollutant loads, achieved the best-cost solution.

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#### **Literature Cited**

- Tchobanoglous G, Burton FL, Stensel HD. Wastewater Engineering: Treatment and Reuse. Metcalf & Eddy, Inc. 4th ed. Boston: McGraw-Hill; 2003.
- Jenkins D, Richard MG, Daigger GT. Manual on the Causes and Control of Activated Sludge Bulking, Foaming and Other Solids Separation Problems. 3rd ed. London, U.K: IWA Publishing; 2004.
- Henze M, van Loosdrecht MCM, Ekama GA, Brdjanovic D. Biological Wastewater Treatment: Principles, Modelling and Design. London, U.K: IWA Publishing; 2008.
- Alva-Argáez A, Kokossis AC, Smith R. Wastewater minimisation of industrial systems using an integrated approach. *Comput Chem Eng.* 1998;22:S741–S744.
- Bandyopadhaya S. Process water management. Ind Eng Chem Res. 2006;45:5287–5297.
- Wang YP, Smith R. Wastewater minimization with flow rate constraints. Chem Eng Res Des. 1995;73:889–904.
- Jödicke G, Fischer U, Hungerbühler K. Wastewater reuse: A new approach to screen for designs with minimal total cost. *Comput Chem Eng.* 2001;25:203–215.
- Prakotpol D, Srinophakun T. GAPinch: genetic algorithm toolbox for water pinch technology. *Chem Eng Proc.* 2004;43:203–217.
- Al-Redhwan SA, Critteden BD, Lababidi HMS. Wastewater minimization under uncertain operational conditions. *Comput Chem Eng.* 2005;29:1009–1021.
- Liu ZY, Li YM, Liu ZH, Wang YJ. A simple method for design of water-using networks with multiple contaminants involving regeneration reuse. AIChE J. 2009;55:374

  –382.
- 11. Wang YP, Smith R. Design of distributed effluent treatment systems. *Chem Eng Sci.* 1994;49:3127–3145.
- Kuo W, Smith R. Effluent treatment system design. Chem Eng Sci. 1997;52:4273–4290.
- Galan B, Grossmann IE. Optimal design of distributed wastewater treatment networks. Ind Eng Chem Res. 1998;37:4036–4048.
- Lee S, Grossmann IE. Global optimization of nonlinear generalized disjunctive programming with bilinear equality constraints: applications to process networks. *Comput Chem Eng.* 2003;27:1557–1575.

- Meyer CA, Floudas CA. Global optimization of a combinatorially complex generalized pooling problem. AIChE J. 2006;52:1027– 1037
- Karuppiah R, Grossmann IE. Global optimization for the synthesis of integrated water systems in chemical processes. *Comput Chem Eng.* 2006;30:650–673.
- 17. Cortés U, Poch M. Advanced agent-based environmental management systems. Whitestein series in software agent technologies. Basel: Birkhäuser Verlag; 2009.
- Verdaguer M, Aulinas M, Clara N, Poch M. Integrating swarm intelligence in a multi-agent system for wastewater discharges management. In: Proceedings of the 5th IEEE International Conference on Intelligent Systems, IS'10. 2010:444

  –449.
- Dorigo M, Maniezzo V, Colorni A. The ant system: optimization by a colony of cooperating agents. *IEEE Trans Syst Man Cybern Part* B. 1996;26:29–41.
- Dorigo M, Stützle, T. Ant colony optimization. Cambridge: MIT Press; 2004.
- Zlochin M, Birattari M, Meuleau N, Dorigo M. Model-based search for combinatorial optimization: a critical survey. *Annals Operat Res*. 2004;131:373–395.

- Gutjahr WJ. ACO algorithms with guaranteed convergence to the optimal solution. *Info Process Lett.* 2002;82:145–153.
- Dorigo M, Di Caro G, Gambardella LM. Ant algorithms for discrete optimization. Artificial Life. 1999;5:137–172.
- Solnon C. Ants can solve constraint satisfaction problems. IEEE Trans Evol Comput. 2002;6:347–357.
- Afshar MH. Penalty adapting ant algorithm: application to pipe network optimization. *Eng Optimiz*. 2008;40:969–987.
- Nahas N, Nourelfath M. Ant system for reliability optimization of a series system with multiple-choice and budget constraints. *Reliab* Eng Syst Safety. 2005;87:1–12.
- Michalewicz Z, Schouenauer M. Evolutionary algorithms for constrained parameter optimization problem. Evol Comput. 1996;4:1–32
- Coello CA. Theoretical and numerical constraint-handling techniques used with evolutionary algorithms: a survey of the state of the art. Comput Methods Appl Mech Eng. 2002;191:1245–1287.
- Government of Catalonia. Diari Oficial Generalitat de Catalunya. December 31. 2008;5288:94876–94877.

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